SILESIAN UNIVERSITY IN OPAVA
Institute of Physics in Opava

Computer modelling of accretion processes in binary systems with black holes and neutron stars

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ABSTRACT OF DOCTORAL THESIS
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Počítačové modelování akrečních procesů v binárních systémech s černými děrami a neutronovými hvězdami

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Výsledky tvořící disertační práci byly získány během doktorského studia ve studijním programu Teoretická fyzika a astrofyzika na Fyzikálním ústavu v Opavě, Slezské univerzity v Opavě.

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Autoreferát byl rozeslán dne ....

Summary

The dissertation is written as an annotated collection of selected papers presenting the results I achieved during my doctoral studies at the Institute of Physics of the Silesian University in Opava. It focuses on the numerical simulation of accretion onto compact objects. I was co-author of six papers published in prestigious international journals and four proceedings papers on this topic. In one paper and two proceeding papers, I was the leading author. One additional paper is submitted for peer review.

The first part of the thesis is devoted to an overview of the topics addressed within the selected papers. Chapters 1-4 describe the astrophysical picture of accreting systems, focusing on the behaviour of X-ray binaries that exhibit effects of the general relativity in combination with radiation magnetohydrodynamic in a complex interplay, introduce complex numerical methods used to model the behaviour of accreting matter and introduce the puffy accretion disk model. Chapter 5 focuses on modelling X-ray variability and quasi-periodic oscillations observed in X-ray binaries.

Part II of the thesis consists of six selected published papers. Papers selected for the annotation are denoted by an asterisk symbol in the complete list of publications and conference contributions at the end of this abstract. This abstract briefly recalls the main points of Chapters 1-5 of the thesis.

Accretion in strong gravity

Black holes (BHs), neutron stars (NSs) or other compact objects are the power behind some of the most luminous objects in our Universe. Their observations across the electromagnetic (EM) spectrum can uncover the properties of the compact object, whose surrounding is governed by the effects of general relativity (GR). The compact objects' extreme environment and surroundings are an excellent laboratory for testing state-of-the-art theories and advancing the understanding of the physical processes and the Universe.

A broad spectrum of accreting compact objects can be observed and studied thanks to advanced observatories monitoring the sky across the whole EM spectrum. Even though the sources are extremely distant and small and thus are often spatially unresolved and appear as point sources, the observations provide valuable insights into these luminous sources' composition, variability, and spectral properties. The lack of spatial resolution makes it all the more challenging to analyse the observed data to obtain as much information about the object as possible. Usually, the spectral energy distribution (SED) and the light curves are the only available data, and using advanced methods and algorithms, the parameters of the compact object can be recovered. However, an essential part of this process is verifying the results using analytical and numerical modelling of the physical processes driving the accretion as the source of radiation, which is crucial.

Binaries with compact objects

One of the brightest sources are the X-ray binaries (XRBs), which typically consist of a compact object and a companion star (Remillard & McClintock 2006, and references therein). The companion can be a dwarf or a main-sequence star in the case of low-mass XRBs (LMXBs), or an O or B type massive star with strong radiation-pressure driven winds in the high-mass XRBs (HMXBs) (Bradt & McClintock 1983). The material from the companion falls onto the compact object due to its strong gravitational pull. Since it usually has a non-zero angular momentum, it forms an accretion disk around the central object. The material in the disk slowly loses its angular momentum, and a large portion of its binding energy is radiated away, especially as

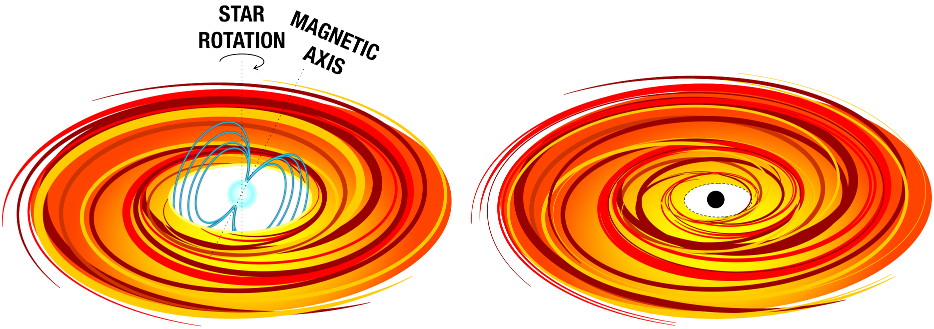

Figure 1. Accretion disk in a NS system (*left*), where the disk is truncated due to the presence of a strong magnetic field, and in a BH system (*right*) where it extends all the way to the innermost stable circular orbit (dashed line).

X-ray, which originates in the innermost regions of the accretion disk. An illustration of a BH LMXB is shown in Figure 2.

The compact object in the XRBs can be a NS, BH, or another exotic object. The NS systems are usually easily distinguishable due to the observation of periodic pulsations or the occasional occurrence of X-ray bursts, which are not observed in BH sources. The pulsations arise from the rotation of the NS and the misalignment between its rotational and magnetic axes (see left panel of Figure 1). NS systems undergo a complex life cycle, from being bright in the radio band to bright in the X-ray band, dimming down and being reborn, and occasionally emitting fast and intense X-ray bursts. Fast time variability, often manifested as high-frequency quasi-periodic oscillations (QPOs), is also more prominent in NS sources compared to BH sources, (Török et al. 2022, and references therein).

In XRBs containing a BH, the source of the radiation is not the BH itself but rather its immediate surroundings and the accretion disk. However, BHs are much more compact than the NSs. Thus, the gravitational curvature of spacetime in its vicinity profoundly impacts the behaviour of matter, leading to observable properties that differ significantly from those expected in a flat spacetime (Bardeen et al. 1972; Frank et al. 2002; Lynden-Bell 1969).

LMXBs predominantly emit radiation in the X-ray band and are relatively faint in other wavelengths. Due to the source of X-rays lying deep in the strong gravitational field of the compact object, studying LMXBs provides an opportunity to obtain valuable insights into the properties and behaviour of matter under extreme gravitational conditions.

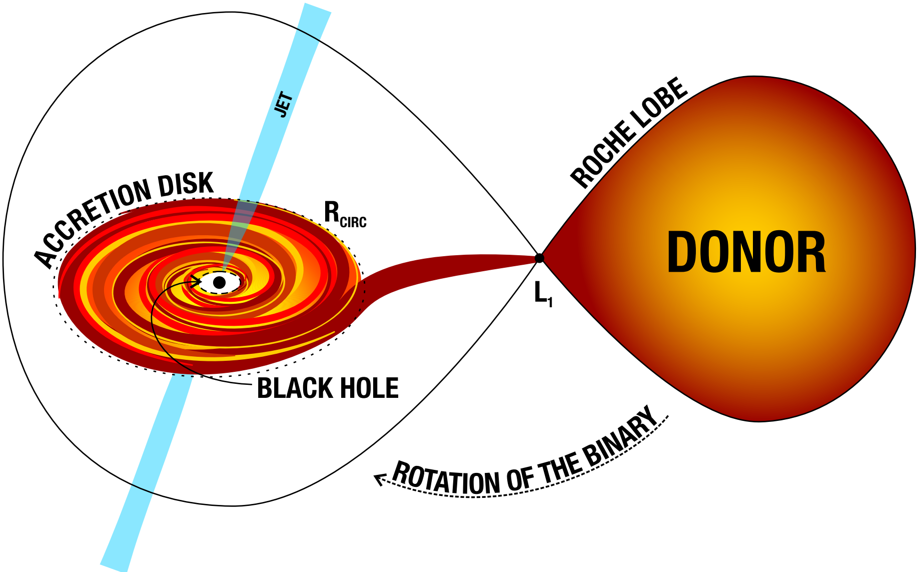

Figure 2. Illustration of typical BH LMXB in a microquasar with radio loud jet. The material flows through the L_1 point from the companion star, which fills up its Roche lobe. The disk's outer edge is at the circularization radius $R_{\rm circ}$, and the inner edge is at the innermost stable circular orbit.

Microquasars

XRBs, where a jet is present and resembles scaled-down quasars, are called microquasars (Mirabel et al. 1992). They are primarily observed in X-rays, and if the jet is formed, they can also be detected in the radio wavelengths. The comparison of observational data obtained from these two bands, which originates from very different physical processes, provides valuable insights into the extreme environment surrounding compact objects. It makes the microquasars a popular target of observations and a hot research topic (see, e.g., Méndez et al. 2022).

BH microquasars undergo irregular outbursts and quiescent periods characterized by distinct spectral and timing properties of the observed signal (Done et al. 2007; Tetarenko et al. 2016). During the quiescent phase, which can last for years, the luminosity of the source is extremely low, making some sources undetectable. However, the spectral and timing properties change rapidly during an outburst, an active period of very high luminosity.

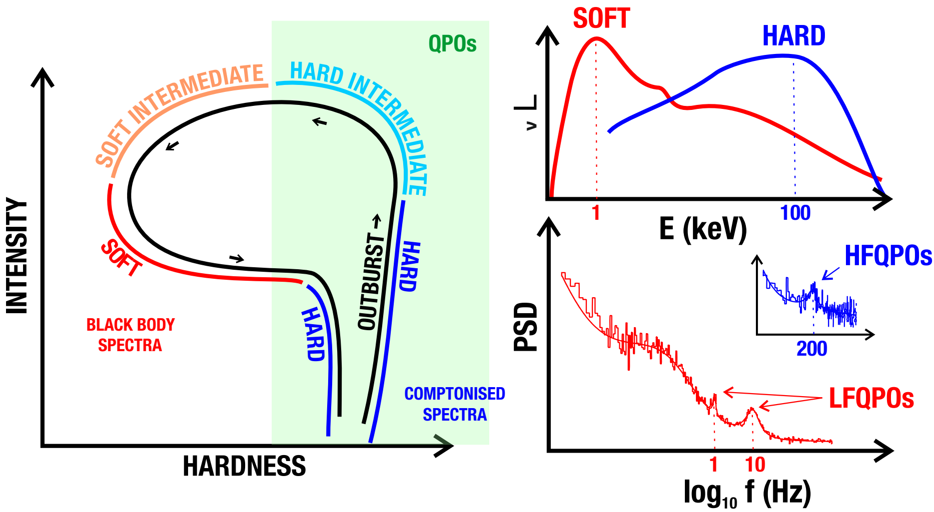

Figure 3. The q curve in the HID of a BH microquasar outburst (left), spectral and timing properties (right). Data for the spectra and PSD from Gierliński et al. (1999) and Motta et al. (2022)

The evolution of the outburst is often represented by a q-shaped curve on a hardness-intensity diagram (HID), where the X-ray intensity is plotted against the observed X-ray hardness ratio (typically $(4-10)/(2-4)\,\mathrm{keV}$). The HID traces the X-ray luminosity and energy spectrum variations throughout the outburst. The HID diagram and properties of the observed signal during different stages are shown in Figure 3.

XRBs also exhibit variability across a wide range of timescales in all wavelengths, with the fastest variation reaching frequencies of units of kHz. The fastest time-variability is observed as the QPOs, manifesting as strong, broad peaks in the power spectral density (PSD) diagram obtained from Fourier analysis of observed X-ray light curves. Several types of QPOs were observed, from low-frequency (LF) QPOs, ranging from units to tens of Hz, to high-frequency (HF) QPOs with frequencies up to units of kHz. The properties of QPOs evolve in conjunction with spectral state transitions in the accretion disk, see Figures 4 and 3.

QPOs originate in the innermost and hottest regions of the accretion disk. However, the exact physical mechanism behind them still needs to be fully understood, although several theoretical models have been proposed. QPOs provide valuable insight into the accretion processes in binary systems. For example, the frequency and amplitude of the QPOs can be used to estimate the mass and spin of a compact object (Goluchová et al. 2019; Kotrlová et al. 2020; Motta et al. 2014;

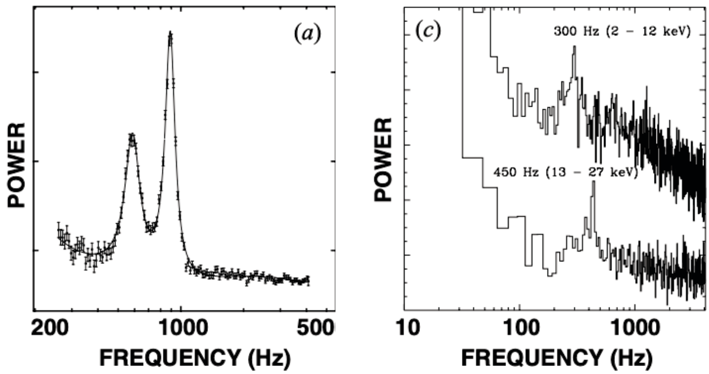

Figure 4. A typical PSD diagram of a NS XRB source, showing the kHz QPOs (*left*) in a NS source and HF QPOs observed in a BH source (*right*). It is obvious that the peaks in the high frequencies are typically weaker in BHs than those of NSs source. Adapted from Van der Klis (2006).

Török et al. 2022) or, in the case of a NS, it can put constraints on the dense matter equation of state, (Urbancová et al. 2019; Urbanec et al. 2013), as well as the size and structure of the accretion disk, and the properties of the surrounding gas. Thus, QPOs can serve as a valuable tool for studying the physics of extreme environments.

Accretion disk

Accretion disks formed around compact objects can be highly luminous and power bright sources. Due to the high temperatures in the innermost area of the disk, which can reach up to 10^8 K, it is mainly in the X-ray band. Consequently, the study of accretion is a hot topic in high-energy astrophysics, particularly when it involves sources with compact objects, especially BHs.

The material in the disk experiences strong gravitational forces from the central object, and the presence of a strong magnetic field can significantly influence the disk's structure, dynamics, and the generation of powerful jets observable in radio wavelengths.

Incorporating these effects into a realistic model of an accretion disk presents a major challenge that requires a multi-disciplinary approach and advanced computational techniques. However, the potential rewards are significant, as a deeper understanding of accretion disks can provide valuable insights into the formation and evolution of BHs and the behaviour of matter under extreme conditions.

The luminosity of an accretion disk is usually scaled by the Eddington luminosity, which can be derived in Newtonian gravity, assuming a spherical accretion onto a point mass M, as

$$L_{\rm Edd} = \frac{4\pi cGM}{\kappa_{es}} = 1.3 \times 10^{38} \left(\frac{M}{\rm M_\odot}\right) {\rm erg \cdot s^{-1}}, \tag{1}$$

where c is the speed of light and G the gravitational constant, $\rm M_\odot$ the solar mass, and κ_{es} the electron scattering opacity.

Analytical models of accretion disks

Analytical solutions for the structure of an accretion disk can be derived based on the energy, mass, and momentum conservation principles. However, in order to obtain a solutions, several assumptions are typically made, for example:

- neglecting self-gravitation effects,
- assuming that the disk midplane lies in the equatorial plane of the central object,
- using the ideal gas approximation,
- assuming a steady and axisymmetric disk,
- neglecting the effects of a large-scale magnetic field,
- assuming hydrostatic equilibrium in the vertical direction,
- assuming Keplerian motion of the fluid.

These assumptions lead to analytical solutions for the disk structure. However, the solutions may not fully capture the complexities of real-world accretion processes, and more sophisticated numerical simulations are often needed for a more accurate description.

Many analytical models of accretion disks have been proposed over the years, corresponding to different accretion regimes, typically characterized by $\dot{m} = \dot{M}/\dot{M}_{\rm Edd}$. These models differ in terms of the optical depth τ, half-thickness ratio to radius H/r, radiative efficiency, or temperature T.

One of the first analytical models of accretion disk was introduced by Shakura & Sunyaev (1973), describing a geometrically thin, optically thick disk. In the same year, Novikov & Thorne (1973) and then Page & Thorne (1974) extended the solution to include the effects of GR, providing corrections for a Kerr BH.

The thin disk model is radiatively efficient, and the inner edge is placed on the innermost stable circular orbit (ISCO) located at radius $r_{\rm ISCO}$, where the boundary condition is such that the stress is zero. The thin disk quickly gained popularity and has been widely used to fit observational data from sources with compact objects (Li et al. 2005a). In this model, the viscosity

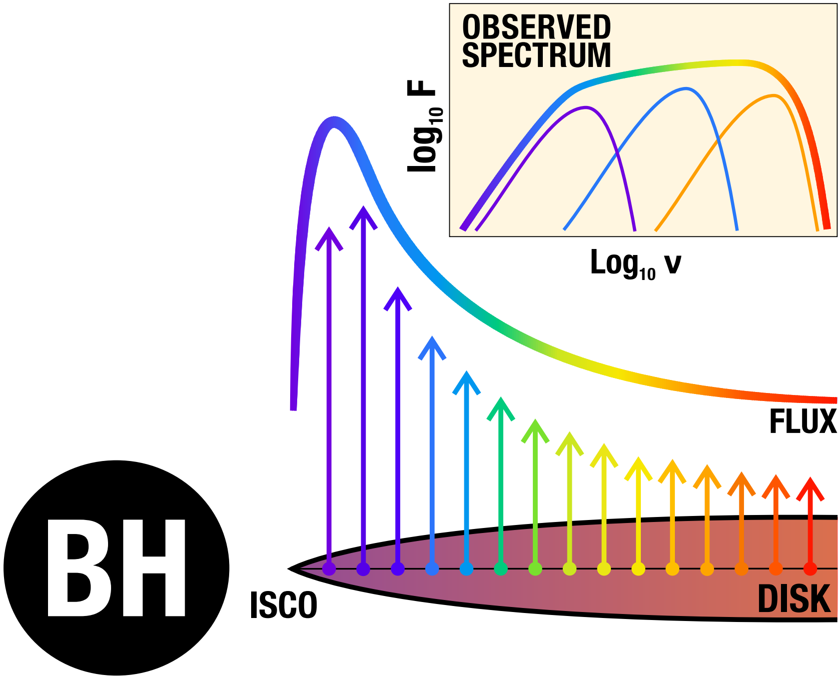

Figure 5. Illustration of a thin disk and its local flux. The inset shows the observed multi-black body spectrum as a sum of black-body spectra corresponding to the temperature on certain radii.

responsible for the transport of angular momentum is parameterized by the viscous coefficient α, the viscous stress $t_{r\phi}$ in the disk is assumed to be proportional to the pressure p,

$$t_{r\phi} = \alpha p. \tag{2}$$

This α parametrization operates well even though the specific source of viscosity remains unknown. For many years, various possibilities were considered to explain the origin of viscosity in astrophysical plasma. It was not until almost 20 years after the thin disk model was constructed that Balbus & Hawley (1991, 1998) introduced the concept of magnetorotational instability (MRI) as a potential source of viscosity.

The MRI is an instability occurring in magnetized rotating systems, such as accretion disks. It is caused by the differential rotation of the ionized gas carrying a low-scale magnetic field. The stretching of magnetic field lines gives rise to turbulences in the accretion flow and carries

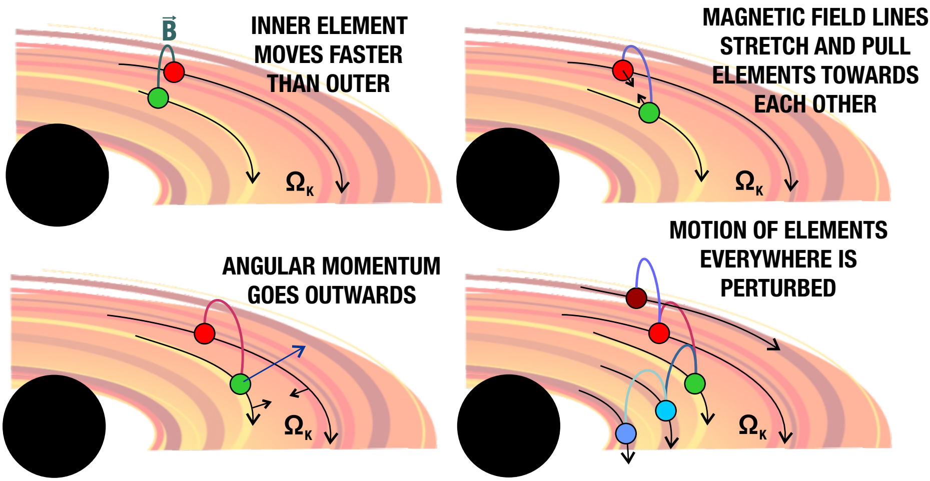

Figure 6. Schematic explanation of the MRI.

the angular momentum outward, even in the case of a very weak magnetic field. Figure 6 shows a schematics of the MRI.

For a very low $\dot{m}$, such as in the case of the BH in the centre of the Galaxy, `Sgr` A*, the accretion disk is optically thin, and the gas density is so low that Coulomb interactions are no longer effective. As a result, the electrons and ions have significantly different temperatures, with the electrons being much colder than the ions (Narayan & McClintock 2008). Radiative cooling is insufficient in this regime, and the dominant cooling mechanism is advection. These disks are often called Advection-Dominated Accretion Flows (ADAFs) and emit a power-law spectrum with a strong Compton component.

As the mass accretion rate increases, the gas density also increases, Coulomb coupling becomes efficient, and the ions and electrons temperatures equal. The disk becomes optically thick, and the radiation cooling becomes efficient. In this regime, the disk flattens into a thin disk of Shakura & Sunyaev (1976). The thin disk emits a multi-temperature black body spectrum, illustrated in Figure 5.

However, once the radiation pressure starts to dominate, the thin disk model is known to be thermally and viscously unstable (Lightman & Eardley 1974; Shakura & Sunyaev 1976). Nevertheless, observations have shown that thin disks can still be observed at luminosities corresponding to much higher mass accretion rates during outbursts when the system is in a soft spectral state (Li et al. 2005b; McClintock et al. 2014).

For $\dot{m} \sim 1$, the advection of radiation (photon trapping) becomes an important cooling mechanism, and this state corresponds to the slim disk model, an (essentially one-dimensional) analytic model of an optically thick advective disk, with $H/r \approx 1$ (Abramowicz et al. 1988).

The slim disk is often used to model the luminous state of XRBs and ultraluminous X-ray sources (ULXs) (Foschini et al. 2006). It is less radiatively efficient than the thin disk since a fraction of the photons is advected into the BH. The inner edge of the slim disk is located under $r_{\rm ISCO}$.

The Eddington limit is interpreted as the maximal luminosity an accreting system can reach, and consequently, the $\dot{M}_{\rm Edd}$ the highest possible mass accretion rate before radiation pressure prevents accretion. However, this value is derived under many assumptions, such as spherical accretion. Higher luminosities can be reached by breaking them, such as considering a thick accretion disk (or torus) with $H/r > 1$. In such a case, a funnel forms around the rotational axis, allowing radiation to escape and reduce radiation pressure acting on the accreting material. The escaping radiation is also highly collimated towards observers close to the rotational axis and obscured by the thick torus body for equatorial observers. Super-Eddington accretion is frequently observed in the Universe, as seen in ULXs, active galactic nuclei, and microquasars.

The most popular model of the super-Eddington accretion disk is the “Polish doughnut” (Abramowicz et al. 1978; Jaroszyński et al. 1980), describing a radiation pressure-supported accretion torus. In the Polish doughnut model, the matter distribution is assumed to be stationary and axially symmetric, and the angular momentum is constant within the disk.

A particular case of the Polish doughnut model is the cusp torus, the largest physical configuration of the accretion tori, where the fluid of the disk fills the largest closed equipressure surface, forming a cusp on its inner edge. The cusp torus is implemented in various QPOs models (Abramowicz & Kluźniak 2001; Blaes et al. 2006; Kluźniak & Abramowicz 2001; Török et al. 2016a, 2022).

Puffy accretion disk

The puffy disk is a model of an accretion disk based on the results of numerical simulation, targeted to study properties and behaviour of sub-Eddington accretion onto a stellar-mass BH, introduced in Lančová et al. (2019).

Simulation of the sub-Eddington accretion can also help to interpret the data obtained from observations of accretion systems, such as microquasars containing a BH. By comparing the simulated observational signatures of the puffy disk with the actual astrophysical data, we can refine our models and gain insights into the physical processes occurring within these systems. Additionally, commonly used tools and methods for analyzing data from observations can be tested against the puffy disk simulations (Lančová et al. 2023; Wielgus et al. 2020).

Global general relativistic radiative magnetohydrodynamical simulations

The global general relativistic (GR) radiative (R) magneto (M) hydrodynamics (HD) method is used to simulate the flow of magnetized plasma, including the exchange of energy and momentum with radiation in the framework of GR (Anile 1989; Komissarov 1999). Astrophysical fluids in accretion disks, primarily composed of hydrogen, usually have high enough temperatures to be ionized ($T \gtrsim 10^4$ K). The presence of free electrons in the plasma leads to the generation of a magnetic field through electric currents, which significantly influences the properties of the flow.

The simulations on which the puffy disk model is based were performed using the `KORAL` code (Chael et al. 2017; Sądowski & Narayan 2015; Sądowski et al. 2014, 2015, 2013, 2017). `KORAL` is a Godunov, finite-difference code with a implicit-explicit solver implemented for the gas-radiation coupling.

GRRMHD equations

`KORAL` solves the conservation equations of mass, momentum, and energy on a fixed grid in an arbitrary metric. The code adopts geometrized units where $G = c = 1$, the gravitational radius

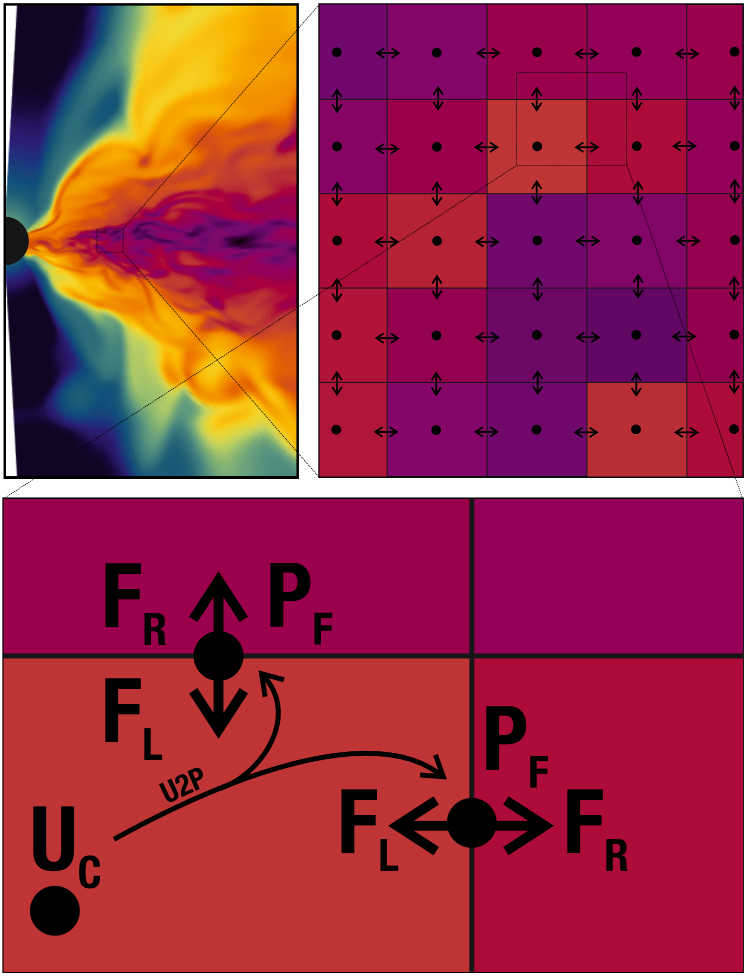

Figure 7. Illustration of the MHD algorithm in 2D. *Top left*: Snapshot of a density map from accretion disk simulation. *Top right*: Zoom in onto the grid, which consists of homogeneous cells. The cell centres and fluxes are indicated. *Bottom*: Zoom-in on a cell boundary, showing the conserved quantities stored at the cell centres, left and right fluxes through the cells' faces, calculated from the primitives reconstructed at the faces. To solve the total flux through each cell face, the Riemann problem is solved.

$r_g = GM/c^2$ as the unit of length, and $t_g = r_g/c$ as the unit of time. Greek indices range from 0 to 3, while Latin indices range from 1 to 3.

The conservation equations for a fluid with a rest mass density ρ and lab frame 4-velocity u^μ are

$$(\rho u^\mu)_{;\mu} = 0, \tag{3}$$

$$(T^\mu{}_\nu)_{;\mu} = G_\nu, \tag{4}$$

$$(R^\mu{}_\nu)_{;\mu} = -G_\nu, \tag{5}$$

$$(F^{*\mu\nu})_{;\mu} = 0, \tag{6}$$

where $R^\mu{}_\nu$ is the radiation stress-energy tensor, G_ν the radiative 4-force density describing the exchange of energy and momentum between the gas and radiation, and $F^{*\mu\nu}$ the dual tensor of the electromagnetic field. Under the ideal MHD approximation used in `KORAL`, assuming infinite conductivity of the fluid, $F^{*\mu\nu}$ is

$$F^{*\mu\nu} = b^\mu u^\nu - u^\mu b^\nu. \tag{7}$$

The gas stress-energy tensor, $T^{\mu\nu}$, is given as

$$T^{\mu\nu} = \left(\rho + u_{\rm int} + p_{\rm gas} + b^2\right) u^\mu u^\nu + \left(p_{\rm gas} + \frac{b^2}{2}\right) g^{\mu\nu} - b^\mu b^\nu, \tag{8}$$

where $u_{\rm int}$ is the internal energy density, $p_{\rm gas}$ is the gas pressure, and $g^{\mu\nu}$ represents the metric tensor.

Radiation

The hot material in accretion disks produces EM radiation, which interacts with the gas and can significantly influence the fluid's dynamics and temperature by emission and absorption. In the case of an optically thick fluid, the photons are effectively trapped and influence the flow dynamics by generating radiation pressure. Photon production in the fluid is also an essential cooling mechanism to be considered when simulating radiatively efficient accretion regimes.

In the `KORAL` code, the fluid and radiation is conservatively exchanging energy and momentum through the radiation 4-force density G^ν, as apparent from equations (4) and (5). G^ν includes exchange through scattering, absorption and photon number conserving Comptonization as introduced in (Sądowski & Narayan 2015; Sądowski et al. 2013).

In addition, the complete form of the radiation tensor $R^{\mu\nu}$ is found using the M1 closure scheme (Levermore 1984; Sądowski et al. 2013). $R^{\mu\nu}$ is constructed from components corresponding to various moments of the frequency-integrated specific intensity. In the fluid frame, it is the radiative energy density $\widehat{E}$, fluxes $\widehat{F}^i$ and the pressure tensor $\widehat{P}^{ij}$. They together form (Mihalas & Mihalas 1984; Sądowski et al. 2013)

$$\widehat{\mathbf{R}} = \begin{pmatrix} \widehat{E} & \widehat{F}^i \\ \widehat{F}^j & \widehat{P}^{ij} \end{pmatrix}. \tag{9}$$

The KORAL code implements advanced numerical methods to solve these equations on a fixed grid, where the quantities' flux through all cells' faces. Additionally, several additional problems must be solved to correctly solve a long-time evolution, such as the dissipation of the magnetic field in an axisymmetric set-up. The illustration of the KORAL algorithm is shown in Figure 7.

The puffy accretion disk

The puffy disk model is a sub-Eddington accretion disk stabilized by magnetic pressure. The simulations were performed for a non-rotating stellar-mass BH of $M = 10\,\mathrm{M}_\odot$ and three values of $\dot{m}$, corresponding to 1.5, 0.9 and 0.6.

Unlike the traditional thin disk model, which assumes a geometrically thin and optically thick structure, the puffy disk consists of a vertically extended and optically thick layer surrounding a geometrically thin core. The puffy disk is stabilized by the magnetic pressure against the thermal and viscous instabilities present in the radiation-pressure domain of the thin disk model.

The disk is both geometrically and optically thick. It can be divided into three distinct regions, each with unique properties:

- Core (C on Figure 8): This region resembles a geometrically thin Keplerian disk of high density, gas temperatures around 10^7 K, and subsonic accretion velocities.
- Puffy region (P on Figure 8): A vertically extended optically thick area of hot magnetized gas.
- Funnel (F on Figure 8): Located above the disk, this optically thin region exhibits strong outflows.

The fluid within the disk shows a high level of magnetization, and the magnetic field is turbulent in the disk; however, these turbulent fluctuations are averaged out over the entire simulation run to establish a magnetic field with the azimuthal component dominating within the disk, while the radial component dominates in the funnel region, as shown in Figure 9.

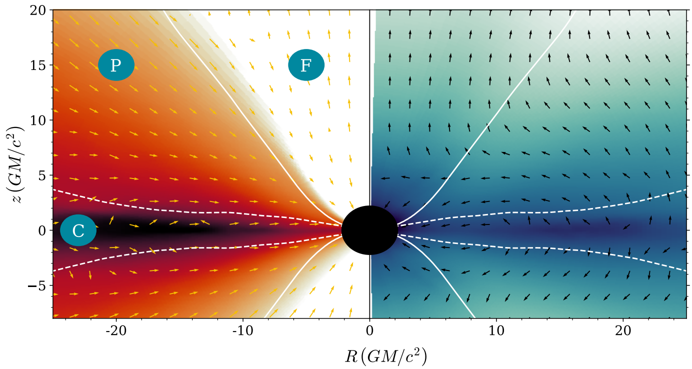

Figure 8. The structure of the puffy disk: Gas and radiation in the $r-\theta$. The full white line represents the photosphere, while the dashed line indicates the density scale height. *Left:* Logarithm of density with denoted regions. Quivers show the gas momentum flux. *Right:* Logarithmic colour map displaying radiation energy density, accompanied by quivers indicating the radiation flux.

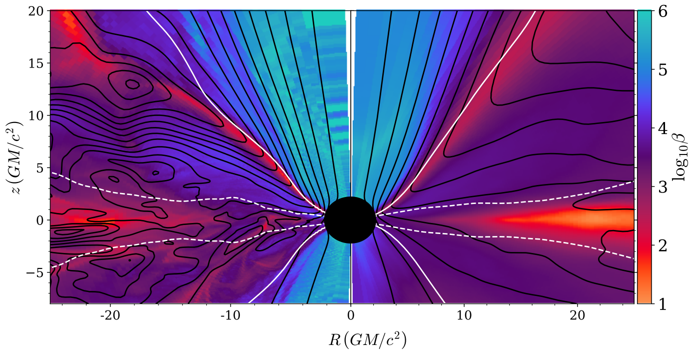

Figure 9. Magnetic field lines (black contours) and magnetization β shown in colour. The white solid line represents the photosphere, while the dashed lines indicate the density scale height. The *right* panel displays data averaged over both azimuthal angle and time. The *left* panel presents a snapshot from the simulation, averaged only over the azimuthal angle.

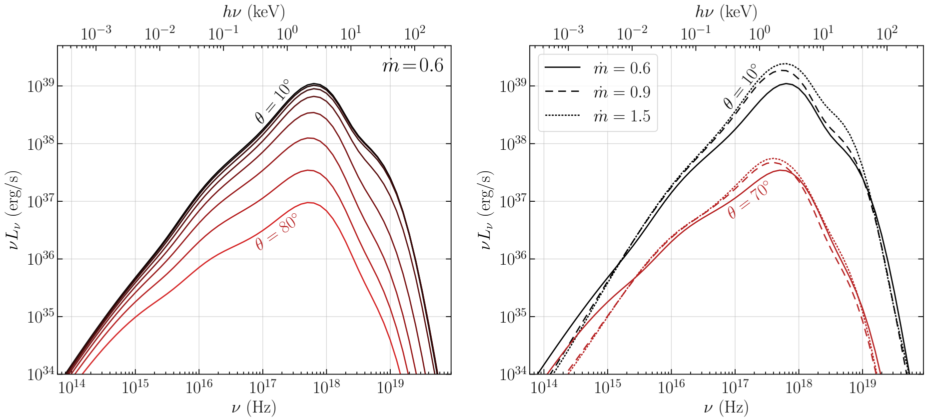

Figure 10. *Left*: isotropic radiative power per logarithmic frequency interval, as a function of the observer's inclination, from $i = 10°$ to $i = 80°$. *Right*: isotropic radiative power per logarithmic frequency interval for three puffy disks simulations, corresponding to different mass accretion rates.

Observational properties of the puffy disk

The analysis of these observational properties of the puffy disk and comparisons with analytical models were presented in Wielgus et al. (2022), and in Lančová et al. (2023) it was extended to include also the `kynbb` spectral model (Dovčiak et al. 2004) in the XSPEC package (Arnaud 1996) and compare it with the `kerrbb` (Li et al. 2005a) and other models.

The puffy disk spectra and images were computed using the relativistic radiative post-processing code `HEROIC` (Narayan et al. 2016; Zhu et al. 2015). This code solves the complete radiative transfer problem on the time-averaged output from the converged simulations for three different mass-accretion rates.

One feature of the puffy disk is the beaming of radiation towards the rotational axis, accompanied by significant obscuration for viewing angles close to the equatorial plane. The inner parts of the disk and the central BH are obscured for observers with high inclinations, see Figure 11. Conversely, for observers with low inclinations, the disk radiation extends all the way to the event horizon.

The spectral energy distribution observed from the puffy disk is similar to that from analytical thin or slim disk models, with a notable additional power-law component corresponding to thermal Comptonization. However, the dependency on the observer's inclination is much stronger than on the $\dot{m}$. While the observer's inclination angle can influence the position of the

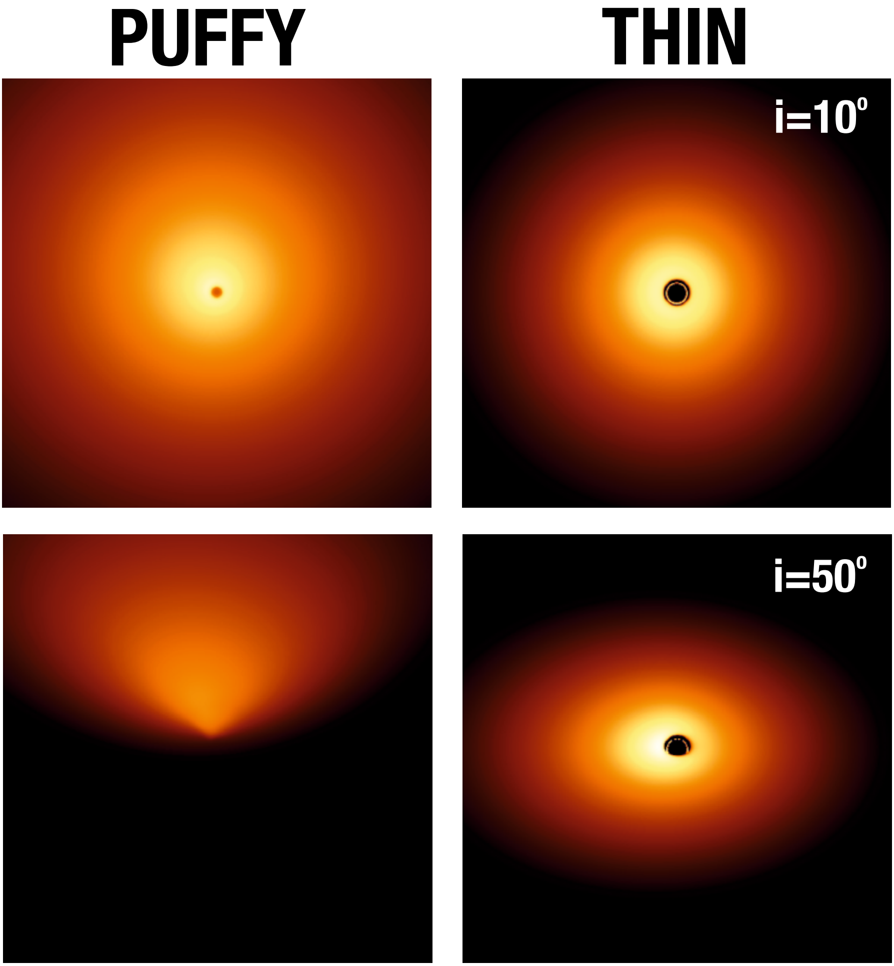

Figure 11. *Left:* Images of the puffy disk, and *right:* images of a standard thin disk with the same $\dot{m}$, for two inclination angles i. The color-map scale is consistent across all images (Wielgus et al. 2022)

maximum luminosity by up to two orders of magnitude, fixing the inclination angle shows no significant change in the SED for different $\dot{m}$ values, as shown in Figure 10. Regardless of the inclination angle, the frequency of photons associated with the peak luminosity remains constant.

Synthetic spectra of the puffy disk were generated for two inclinations of the `S06` simulation to test the performance of commonly used spectral fitting models. These spectra were then fitted using the `XSPEC`, simulating the observation of a real astrophysical source.

The BH mass, distance, viscous α, and inclination angle were fixed for the fitting, and total luminosity, spin, temperature of the seed and up-scattered photons, and photon index were free parameters. The results showed that the puffy disk could be fitted with standard tools and provide a reasonable set of parameters. However, the results of the fitting process overestimated the BH spin compared to the value used in simulations.

Modelling the rapid time variability of XRBs

Observations of bright objects in the X-ray sky reveal distinct properties of these sources, such as the quasi-periodic temporal variability across a wide range of frequencies, the QPOs. Among them, the QPOs of high frequencies (hundreds of Hz or units of kHz) are the most interesting, and at the same time, their modelling needs to explain their origin sufficiently.

The frequencies of HF QPOs are associated with the fundamental frequencies of motion in the innermost regions of accretion disks (Karas 1999a,b; Stella & Vietri 1999; Török et al. 2005). These frequencies are linked to the properties of the central compact object, such as the mass, rotational frequency, or, in the case of the NS source, the star's radius. The HF QPOs were first observed in NS sources, where they are usually labelled as the twin peak QPOs, and they were also identified in BH sources (see Figure 4).

Over the years following their discovery, several models have been proposed to explain the QPOs. These models are founded on the premise that the QPOs originate in the innermost regions of an accretion disk near the central compact object. While each model can explain specific observed properties, they often have limitations when addressing other aspects (Van der Klis 2006).

Foundations of the orbital models of QPOs

The amplitude of the oscillations is much higher in the NS sources compared to the BH systems. The observed frequencies of these oscillations closely align with the fundamental frequencies of orbital motion in the vicinity of a compact object. This alignment is particularly evident for geodesic circular motion with Keplerian orbital frequency

$$f_K = \frac{c^3}{2\pi GM}\frac{1}{a+r^{3/2}}, \tag{10}$$

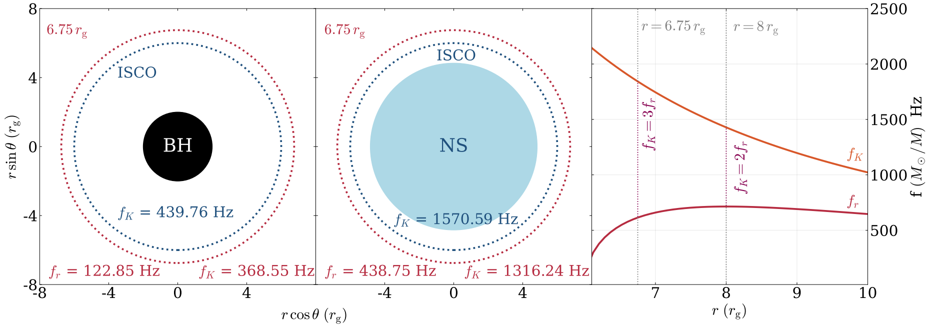

Figure 12. *Left and Middle*: Illustration of a non-rotating BH and NS, accompanied by the corresponding values of Keplerian (f_K) and radial epicyclic (f_r) frequencies at $r_{\rm ISCO}$ and $r = 6.75\, r_{\rm g}$, where $f_K = 3 f_r$. Values for the BH are calculated for $M = 5\,{\rm M}_\odot$, while those for the NS for $M = 1.4\,{\rm M}_\odot$ and radius $R_* = 10$ km. *Right*: Values of the f_K and f_r in the non-rotating spacetime, where also $f_\theta = f_K$.

and the radial f_r and vertical f_θ epicyclic frequencies in the Kerr metric are

$$f_r^2 = f_K^2 \left[\left(1 - \frac{6 r_{\rm g}}{r} \right) + 8a \left(\frac{r_{\rm g}}{r} \right)^{3/2} - 3a^2 \left(\frac{r_{\rm g}}{r} \right)^2 \right], \tag{11}$$

$$f_\theta^2 = f_K^2 \left[1 - 4a \left(\frac{r_{\rm g}}{r} \right)^{3/2} + 3a^2 \left(\frac{r_{\rm g}}{r} \right)^2 \right], \tag{12}$$

where a is the non-dimensional spin parameter of the central object and r is the radius. The radial dependence of these frequencies and their typical values for a stellar-mass BH and a NS are illustrated in Figure 12.

Oscillations of accretion tori

A connection between the QPOs and oscillations of accretion tori have been suggested in many papers, (e.g., Abramowicz et al. 2006; Abramowicz & Kluźniak 2001; Blaes et al. 2006; Bursa 2005; Bursa et al. 2004; de Avellar et al. 2017; Fragile et al. 2016; Ingram & Done 2010; Mishra et al. 2017; Rezzolla et al. 2003).

The accretion tori are often approximated using the Polish doughnut model, which assumes a stationary perfect fluid in purely azimuthal motion. It forms within the surfaces of constant potential, in which the gravitational forces acting on the torus and the internal pressures of the torus are combined (Abramowicz et al. 1978; Jaroszyński et al. 1980).

The cusp-torus model

The cusp torus configuration is promising for modelling oscillation modes in real accretion disks, as it corresponds to a marginally overflowing torus. Oscillations of such torus, situated in the innermost region of an accretion flow, can modulate the amount of matter transferred onto the NS surface, which can be reflected in the luminosity of the boundary layer (Horák 2005; Paczynski 1987).

The cusp-torus (CT) model, initially introduced and preliminary tested on observed data from NS sources in Török et al. (2016b), is based on the solution of Straub & Šrámková (2009) for the eigenfrequencies of torus oscillations in GR. The model was applied to observed QPOs in Galactic microquasars and compared to other models in Kotrlová et al. (2020).

More recently, the CT model has been extended into the Hartle-Thorne metric (Hartle & Thorne 1968; Matuszková et al. 2022; Šrámková et al. 2023), incorporating the influence of the quadrupole moment of a NS, $q = Q/M^3$. This parameter significantly affects the frequencies of orbital motion (Urbancová et al. 2019; Urbanec et al. 2013).

Oscillations of accretion tori and estimations of parameters of compact objects

The frequencies of observed QPOs can be used to derive the parameters of the central compact object. Values deduced from such fitting can be directly compared with outcomes obtained from alternative parameter estimation methods, including those based on the spectral properties of the signal. This comparison helps to validate both the results and methodologies.

The spectral-based and timing-based methods are compared in Figure 13 on selected BH sources, where in three of them, the HF QPOs were observed. The estimations of spin from spectral-based and timing-based methods are compared, showing agreement in some cases. A detailed description of these models and the spin estimation process can be found in Kotrlová et al. (2020).

Figure 13 shows that specific spectral and timing-based methods often yield consistent results, pointing to relatively high spin values in the BH XRBs. However, it is evident that no universally preferred method exists, and the applied methodology remains premature.

In the case of NS sources, the Hartle-Thorne metric within the CT model provides a sufficient approximation for its application on sources with rotating NS. The mass of several atoll sources obtained using the CT model in Török et al. (2022) aligns with estimates obtained from X-ray burst measurements, which is illustrated in Figure 14.

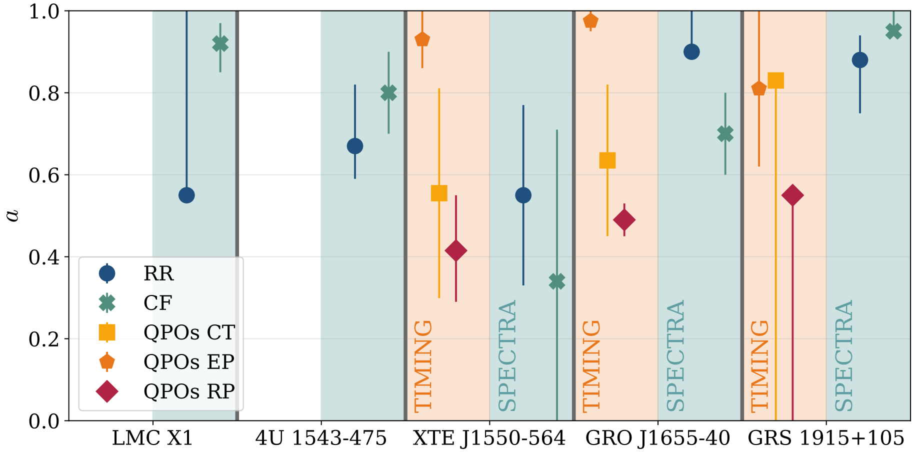

Figure 13. Comparison of spin value estimations for selected BH XRBs. The values obtained with methods based on the spectral properties are in the blue segments of the plots. Values derived through the timing-based methods are in the orange segments of the plots. Data from Kotrlová et al. (2020) and Reynolds (2021).

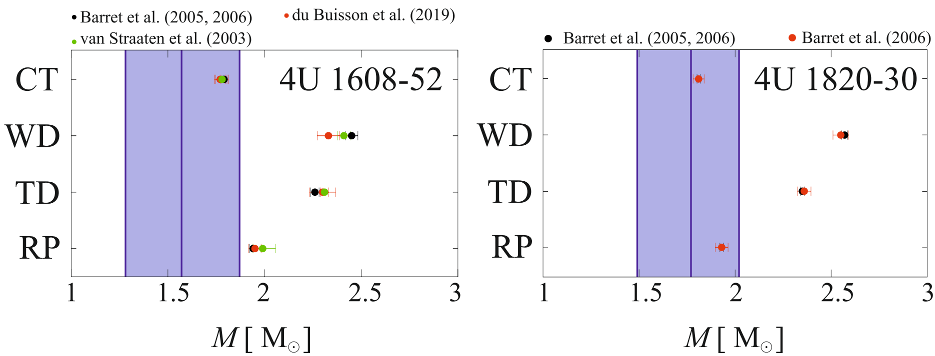

Figure 14. Estimation of the mass for two NS XRBs, comparing the mass derived from type I X-ray bursts (blue rectangles) against results from several QPO models. The WD and TD models are described in Török et al. (2019). Kateřina Klimovičová (private communication), and Török et al. (2019, 2022).

Oscillations of accretion tori and QPO modulation

The oscillations of an accretion torus in the vicinity of a compact object manifest as observable variations in the total flux. Frequencies of these oscillations can then be identified in the Fourier image of the observed light curve.

However, the propagation of light close to a compact object is governed by the effects of GR. To achieve a realistic representation, it is necessary to numerically solve the null geodesic equations determining the trajectories of photons. This can be accomplished using a relativistic ray-tracing code, which accurately simulates the paths of photons in curved spacetimes (Bakala et al. 2015; Beckwith & Done 2005; Bronzwaer et al. 2018; Chan et al. 2013; Cunningham & Bardeen 1973; Dexter & Agol 2009; Karas et al. 1992; Prather et al. 2023; Schnittman & Rezzolla 2006; Vincent et al. 2011).

Within our research group, the Lensing Simulation Device (`LSD`, Bakala et al. 2015, 2007) was developed and applied to explore the effect of light bending and Doppler shift on the observed timing and spectral properties. The `LSD` code is also valuable in modelling the interactions between multiple objects in the strong gravitational fields, accounting for the mutual obscuration effects (Bakala et al. 2014; De Rosa et al. 2019; Feroci et al. 2022; Zhang et al. 2016). The capabilities of the `LSD` code are illustrated in Figure 15.

For the QPO modelling, the `LSD` code can simulate an object in the vicinity of a compact object and its time evolution, producing individual spectra and spatially resolved pictures. However, the code does not solve the radiative transfer through the material, thus, the effects on the spectral shape and distribution arise solely from the GR effects.

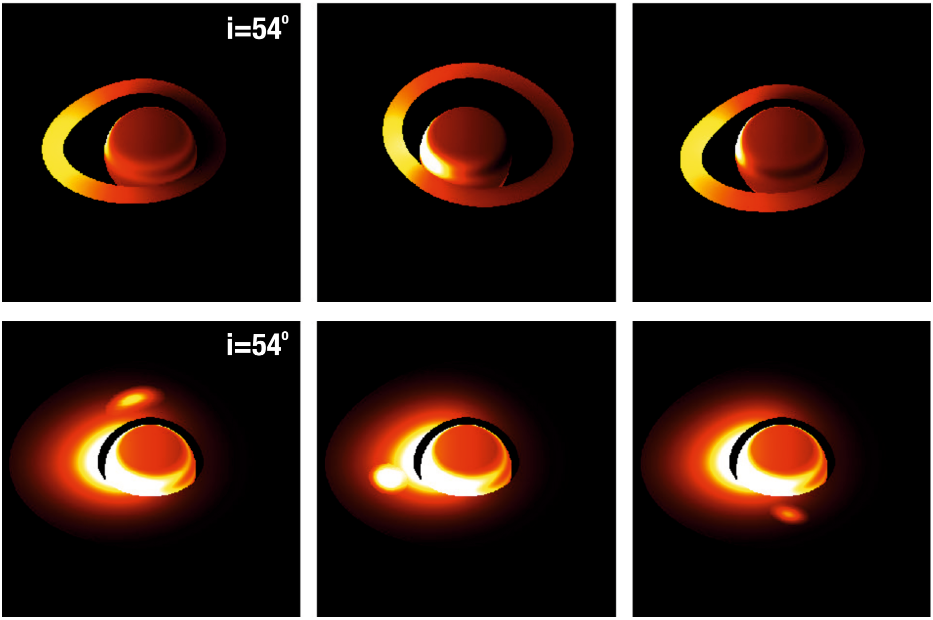

Figure 15. Colourmaps illustrating the evolution of the relative bolometric intensity of the observable radiation for models of oscillating torus (*top*) and a hot spot orbiting in an accretion disk (*bottom*), simulated using the `LSD` code.

Full list of papers published in international journals

- Wielgus, M., Lančová, D., Straub, O., Kluźniak, W., Narayan, R., Abarca, D., Różańska, A., Vincent, F., Török, G., & Abramowicz, M.; Observational properties of puffy discs: radiative GRMHD spectra of mildly sub-Eddington accretion (2022), Mon. Not. R. Astron. Soc., 514, 780, DOI: 10.1093/mnras/stac1317*
- Török, G., Kotrlová, A., Matuszková, M., Klimovičová, K., Lančová, D., Urbancová, G., & Šrámková, E.; Simple Analytic Formula Relating the Mass and Spin of Accreting Compact Objects to Their Rapid X-Ray Variability (2022), Astrophys. J., 929, 28, DOI: 10.3847/1538-4357/ac5ab6*
- Kotrlová, A., Šrámková, E., Török, G., Goluchová, K., Horák, J., Straub, O., Lančová, D., Stuchlík, Z., & Abramowicz, M. A.; Models of high-frequency quasi-periodic oscillations and black hole spin estimates in Galactic microquasars (2020), Astron. Astrophys., 643, A31, DOI: 10.1051/0004-6361/201937097*
- Bakala, P., De Falco, V., Battista, E., Goluchová, K., Lančová, D., Falanga, M., & Stella, L.; Three-dimensional general relativistic Poynting-Robertson effect. II. Radiation field from a rigidly rotating spherical source (2019), Phys. Rev. D, 100, 104053, DOI: 10.1103/PhysRevD.100.104053.
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List of proceedings and submitted papers

- Lančová, D., Yilmaz, A., Wielgus, M., Dovčiak, M., Straub, O., & Török, G.; Spectra of puffy accretion discs: the kynbb fit (2023), Astronomische Nachrichten, 344, e20230023, DOI: 10.1002/asna.20230023*
- Šrámková, E., Matuszková, M., Klimovičová, K., Horák, J., Straub, O., Urbancová, G., Urbanec, M., Karas, V., Török, G., & Lančová, D.; Oscillations of fluid tori around neutron stars (2023), Astronomische Nachrichten, 344, e20220114, DOI: 10.1002/asna.20220114.
- Matuszková, M., Klimovičová, K., Urbancová, G., Lančová, D., Šrámková, E., & Török, G.; Oscillations of non-slender tori in the external Hartle-Thorne geometry (2022), arXiv e-prints, arXiv:2203.10653, DOI: 10.48550/arXiv.2203.10653*

- Lančová, D., Bakala, P., Goluchová, K., Falanga, M., De Falco, V., & Stella, L.; The study on behaviour of thin accretion disc affected by Poynting-Robertson effect (2017), RAGtime 17-19: Workshops on Black Holes and Neutron Stars, 127.
- Karas, V., Klimovičová, K., Lančová, D., Svoboda, J., Török, G., Matuszková, M., Šrámková, E., Šprňa, R., & Urbanec, M.; Timing of accreting neutron stars with future X-ray instruments: towards new constraints on dense matter equation of state (2023), submitted to Contrib. Astron. Obs. Skalnaté Pleso

Presentations at international conferences and invited seminars

- RAGtime 20, Opava, Czech Republic; October 2018; *GRRMHD simulation of thin accretion disk stabilized by magnetic field*, talk
- Heraeus Seminar on Accretion in Strong Gravity, Wilhelm und Else Heraeus Stiftung, Bad Honnef, Germany; February 2019; *Global GRRMHD simulation of thin accretion disk*, talk
- 12th Integral Conference, Geneva, Switzerland; February 2019; *Global GRRMHD simulation of thin accretion disk*, poster
- RAGtime 21, Opava, Czech Republic; September 2019; *Puffy accretion disks: sub-Eddington, optically thick, and stable*, talk
- Future of X-ray Timing 2019, Amsterdam, Netherlands; October 2019; *Global GRRMHD simulation of thin accretion disk*, talk
- BHI colloquium, Harvard University, USA; December 2019; *Puffy accretion disks: sub-Eddington, optically thick, and stable*
- RAGtime 23, Opava, Czech Republic; September 2021; *Puffy accretion disk: observational properties and inner structure*, talk
- 9th Microquasar Workshop, Cagliari, Italy; September 2021; *Puffy accretion disk: sub-Eddington, optically thick, and stable*, poster
- Seminar, NORDITA, Stockholm, Sweden; November 2021; *Puffy accretion disks: new results and plans*
- Seminar, Institute of Astronomy, Cambridge University, Cambridge, United Kingdom; November 2021; *Global GRRMHD simulation of stable, optically thick subEddington accretion disk – the Puffy disk*
- Seminar, INAF Sicily, Palermo, Italy; March 2022; *Puffy accretion disk: subEddington, optically thick, and stable*
- Pharos conference, Rome, Italy; May 2022; *Broadened iron lines exhibited by accreting relativistic compact object*, poster

- XMM-Newton meeting, Black hole accretion under the x-ray microscope, Madrid, Spain; June 2022; *Observational properties of puffy accretion disk*, poster
- Ten Years of High-Energy Universe in Focus - NuSTAR 2022, Cagliari, Italy; June 2022, *Numerical model of a stable sub-Eddington accretion disk – the puffy disk*, talk
- COSPAR 2022, Athens, Greece; July 2022; *Observational properties of puffy accretion disk*, talk
- 31st Texas Symposium on Relativistic Astrophysics, Prague, Czech Republic; September 2022; *GRRMHD simulation of sub-Eddington accretion onto stellar mass black hole*, talk
- RAGtime 23, Opava, Czech Republic; October 2022; *GRRMHD simulation of sub-Eddington accretion onto stellar mass black hole*, talk
- Third Athena Scientific Conference, Barcelona, Spain; November 2022; *Observational properties of puffy accretion disk*, poster
- Timescales in Astrophysics Conference, Abu Dhabi, UAE; January 2023; *Timescales of BH accretion disks*, talk/poster
- HEAD meeting, Waikoloa Village, Hawaii, USA, March 2023; *Puffy accretion disks: sub-Eddington, optically thick, and stable*, poster
- IBWS 2023, Karlovy Vary, Czech Republic; May 2023; *Puffy accretion disks: sub-Eddington, optically thick, and stable*, poster

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Co-authorship statement

I hereby declare that my contribution to the papers included in the thesis was mainly based on independent analytical and numerical calculations, graphical representations of the results, and the preparation of the manuscripts.

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Debora Lančová

The co-authors of the publications included in the thesis gave their permission to use any parts of the papers and any reported results in the submitted thesis. They confirmed the proportional contributions of the authors to the papers.

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In Opava, September 2023